

## Appendix B. SAFIR Sensitivity

### Background

The ultimate performance limit for a cold telescope at L2 is the photon noise from the diffuse astrophysical backgrounds. This Appendix tabulates the intensities of these backgrounds, the astrophysical capability of SAFIR if operating at these limits, and the requirements for detectors which will enable this performance.

### Introduction

Although far-IR – mm-wave astronomy has now been developing for several decades, access to this portion of the spectrum has remained difficult and limited. Relative to their optical-wavelength counterparts, the existing submillimeter observatories are both small in units of wavelength and hot units of photon energy, which imposes limits on their sensitivity. The single-aperture far-IR observatory (SAFIR) offers the potential to overcome these limitations, with its large (10 m), cold (4.5 K) aperture in space. The fundamental sensitivity limit for a cold far-IR space telescope such as SAFIR is the photon noise in the diffuse astrophysical backgrounds. If limited only by these backgrounds, SAFIR has the potential for 4–5 orders of magnitude sensitivity improvement relative to existing and planned facilities. It is therefore important to estimate the photon noise from these diffuse backgrounds, to anticipate the types of astrophysical experiments that SAFIR will enable and to estimate requirements on the detectors to take full advantage of the low-background environment.

### Photon-Limited Sensitivity

In this analysis, we describe the photon field in terms of orthogonal spatial modes, and assume that these modes are uncorrelated throughout the optical train. In this case, the noise from each mode coupling to a detector adds in quadrature to produce the total noise. This approach is applicable, for example, to diffraction-limited imagers and spectrometers, in which a single detector couples only one or two polarization modes from the sky and telescope. The noise due to photon rate fluctuations is most easily expressed in terms of the effective photon occupation number at the detector,  $\langle n_e \rangle$ .  $\langle n_e \rangle$  is the number of photons occupying a given mode, and in general is a function of frequency. In a small bandwidth  $\delta\nu$ ,  $\langle n_e \rangle$  determines the mean photon rate  $\langle n_e \rangle \delta\nu$ , and corresponding power  $\langle n_e \rangle h\nu \delta\nu$  in this mode. In calculating  $\langle n_e \rangle$ , for a given mode at a given detector, it is important to sum contributions from all sky, telescope, and instrument components, each term weighted by the total coupling from its source through the detection process  $\eta$ :

$$\langle n_e \rangle = \sum \langle n_i \rangle \eta_i \quad (1)$$

For thermal sources described with a temperature  $T$  and emissivity  $\epsilon$ ,  $\langle n \rangle = \epsilon (e^{h\nu/kT} - 1)^{-1}$ . Because photons are bosons, the noise arises from the fluctuations in this rate about its mean value, in a finite bandwidth and integration time  $\tau$ , we have (for a rigorous derivation, see Zmuidzinas (2003 Applied Optics, 42, 4989).

$$\Delta \langle n_{e, \text{rms}} \rangle^2 = (1/\tau) \int_\nu d\nu \langle n_e \rangle [\langle n_e \rangle + 1] \quad (2)$$

Under our assumption of at most a few uncorrelated modes illuminating each detector, we convert the mode occupation into an uncertainty in the measured power in an integration time  $\tau$ :

$$\sigma^2 = (1/\tau) \sum_{\text{modes } m} \int_\nu d\nu (h\nu)^2 \langle n_{e,m} \rangle (\langle n_{e,m} \rangle + 1) \quad (3)$$

is the uncertainty in the measurement, but it depends on the integration time  $\tau$ , which is removed to generate the time-independent noise equivalent power (NEP):

$$\begin{aligned} \text{NEP}_{\text{photon}}[\text{W/Hz}^{1/2}] &= \sqrt{2} \sigma (\tau = 1\text{s}) \\ &= \sqrt{2} \left( \sum_{\text{modes } m} \int_{\nu} d\nu (h\nu)^2 \langle n_{e,m} \rangle (\langle n_{e,m} \rangle + 1) \right)^{1/2} \end{aligned} \quad (4)$$

The utility of this report is to estimate the occupation numbers of the sources that generate the photon noise. Inspection of Eq. 4 shows the two regimes of photon noise: 1) for  $\langle n_e \rangle < 1$ , corresponding to  $h\nu/kT > 1$ , the photons are uncorrelated and the noise scales as the square root of the power on the detector, called “shot noise,” 2) for  $\langle n_e \rangle > 1$ , ( $h\nu/kT < 1$ ), the noise scales as the total power. With a cold space telescope, this transition occurs at  $\lambda \sim 1$  mm: in the mid and far-IR, the zodi and cirrus background has low occupation number, but beyond 1 mm, the background is the optically thick CMB with large occupation number.

In addition to the photon noise, other sources of noise which are not correlated with the photon stream add in quadrature to get the total NEP referred to the detector  $\text{NEP}_{\text{tot}}$ .

$$\text{NEP}_{\text{tot}}^2 = \text{NEP}_{\text{photon}}^2 + \text{NEP}_{\text{detector}}^2 \quad (5)$$

For simplicity in this report, we neglect any contribution from the detectors or amplifiers to the noise budget, thus the sensitivity estimates represent a lower limit. To calculate an astronomical sensitivity, the instrument / detector coupling and the size and efficiency of the aperture must be taken into account:

$$\text{NEF}_{\text{sky}} = \text{NEP}_{\text{det}} / (\eta_i A_{\text{tel}} \eta_A), \quad \text{NEFD}_{\text{sky}} = \text{NEF}_{\text{sky}} / \Delta\nu \quad (6)$$

where the product  $\eta_A A_{\text{tel}}$  is the effective aperture of the telescope for coupling plane wave radiation to the focus. For SAFIR we assume an aperture efficiency of 75%, and an instrument transmission of 25%, and the results presented here can be scaled according to the above equation.

### The Background Sources

We turn now to the backgrounds themselves. The far-IR and submillimeter background was measured with the DIRBE instrument on COBE, and substantial effort has gone into extracting the foreground components to reveal the cosmic extragalactic component. For the purposes of estimating the sensitivity of SAFIR and other future observatories, we need only to use these DIRBE extractions. The intensity of the diffuse emission is typically expressed in MJy  $\text{sr}^{-1}$ , the conversion to an occupation number requires dividing by the photon energy  $h\nu$  and the number of modes per area per solid angle  $2/\lambda^2$ . The contributions of the various backgrounds are presented in Table 1, using DIRBE measured values for the zodi and cirrus out to 240  $\mu\text{m}$ , with extrapolations beyond.

**Zodiacal Light:** From the near-IR through 100–200  $\mu\text{m}$ , the solar system dust scatters solar radiation and emits thermally at  $T \sim 250\text{--}300$  K. Emission from this component peaks at 25  $\mu\text{m}$  and is the brightest background source for an observer in the inner solar system. Relative to the galactic and cosmic backgrounds, the solar-system dust is distinguished by its modulation with the earth’s orbit. Kelsall et al. (1998 *Astrophys. J.* 508, 44) present the DIRBE maps, we extract

intensities toward the ecliptic poles (low zodi), representing the lowest possible backgrounds, appropriate for 10% of the sky. The plots presented in this report are appropriate for the low-zodi sightlines. More typical values are also tabulated (medium zodi), appropriate for directions off the poles but not in the plane. It is straightforward to scale the sensitivity to these lines of sight as the square root of the occupation number. For wavelengths beyond 240  $\mu\text{m}$  we extrapolate by assuming 280 K dust with a  $\nu^{1.5}$  emissivity law.

**Table 1: Diffuse background sources for a cold telescope at 1 A.U.**  
Photon Occupation Number

[ $\mu\text{m}$ ]	Low Zodi	Int. Zodi	Low Gal.	Int. Gal.	CMB	4.5 K	10 K	20 K
12	8.7 (-11)	4.9 (-10)	1.1 (-11)	4.3 (-11)				
25	1.6 (-9)	7.0 (-9)	2.9 (-10)	5.9 (-10)				1.6 (-14)
60	3.7 (-8)	4.8 (-8)	1.1 (-8)	6.1 (-9)			1.9 (-9)	3.1 (-7)
100	1.3 (-7)	2.2 (-7)	5.0 (-8)	1.9 (-7)			2.8 (-8)	3.8 (-5)
140	1.6 (-7)	1.3 (-6)	1.6 (-7)	6.2 (-7)			1.7 (-6)	3.0 (-4)
240	3.5 (-7)	4.2 (-6)	6.4 (-7)	3.5 (-6)	2.9 (-10)	8(-8)	1.3 (-4)	2.6 (-3)
400	2.8 (-7)	3.4 (-6)	1.1 (-6)	6.1 (-6)	1.9 (-6)	1.7 (-5)	1.4 (-3)	9.9 (-3)
600	2.35 (-7)	2.9 (-6)	1.3 (-6)	7.2 (-6)	1.5 (-4)	2.4 (-4)	5.0 (-3)	2.1 (-2)
1000	1.85 (-7)	2.3 (-6)	1.4 (-6)	7.4 (-6)	5.2 (-3)	2.1 (-3)	1.5 (-2)	4.7 (-2)

**Galactic Cirrus:** At longer wavelengths, as the solar-system dust emission decreases, the much larger column density of cooler ( $T \sim 17\text{--}21$  K) dust from the galaxy dominates. Arendt et al. (1998 *Astrophys. J.* 508, 74) present the maps of this galactic component based on the DIRBE observations. As with the zodi, we extract intensities toward the holes (low cirrus) as well as more typical intensities appropriate for directions out of the plane of the galaxy, but plot the sensitivity appropriate for the low cirrus directions. Extrapolation beyond 240  $\mu\text{m}$  assumes 20 K dust with  $\nu^{1.5}$  emissivity.

**Telescope and CMB:** The telescope and microwave background (CMB) are straightforward thermal sources. For the telescope, we consider several temperature values and assume a conservative emissivity of 5%. This is larger than the emissivity of the mirror surface itself, to account for loading from obstructions and edges. The CMB is a 2.73 K source with unit emissivity.

### Observatory Sensitivity

Using power-law interpolations of the background sources presented in Table 1, we plot the sensitivities achievable with SAFIR under photon-noise-limited conditions. The first plot (Figure 1) shows the photon noise at the detector for both continuum observations and spectroscopy, expressed as a noise equivalent power (NEP). This is an indication of the detector sensitivity required to reach the photon noise limit. While a discussion of the various detector technologies and their capabilities is beyond the scope of this Appendix, we point out that the requirements for background-limited spectroscopy are beyond what is currently achievable, see Young et al. (2002 *Detector Needs for Long-Wavelength Astrophysics*, NASA Report) for a detailed discussion.

Figure 2 plots the SAFIR's sensitivity in the continuum, along with that current and future far-IR submm platforms. SAFIR offers the potential for  $10^4\text{--}10^5$  orders of magnitude improvement in raw sensitivity, which translates to  $10^8\text{--}10^{10}$  times faster mapping for a given array size and map depth. For very deep observations of the extragalactic sky at the longer wavelengths, source confusion will quickly become the most important limitation for SAFIR. At the shorter

wavelengths ( $<100\ \mu\text{m}$ ), the source densities will remain high, but the confusion limit allows a very deep map. For example, Dole et al. (2004 *Astrophys. J. Supp. Ser.* 154, 93-96) predict a confusion limit of  $4\ \mu\text{Jy}$  at  $70\ \mu\text{m}$ , and that the sources extracted mapping to this depth will represent 99% of the total far-IR background. In brief, for its shorter wavelength bands, SAFIR will provide the angular resolution necessary to resolve the bulk of the far-IR background galaxies, and the raw sensitivity to do so rapidly over large areas of sky. Figure 3 plots the sensitivity of SAFIR for moderate-resolution ( $R=1000$ ) spectroscopy. Because the detector NEPs required for the background-limited performance are beyond current capability, the sensitivity using devices with  $\text{NEP} = 3 \times 10^{-19}\ \text{W}\cdot\text{Hz}^{-1/2}$  is also plotted. The sensitivities of other platforms are also plotted, again illustrating the huge gains possible with the large cold telescope. Unlike in the continuum, source confusion is not a problem for spectroscopy, as any given source has only 1 spectral line per many spectral resolution elements.

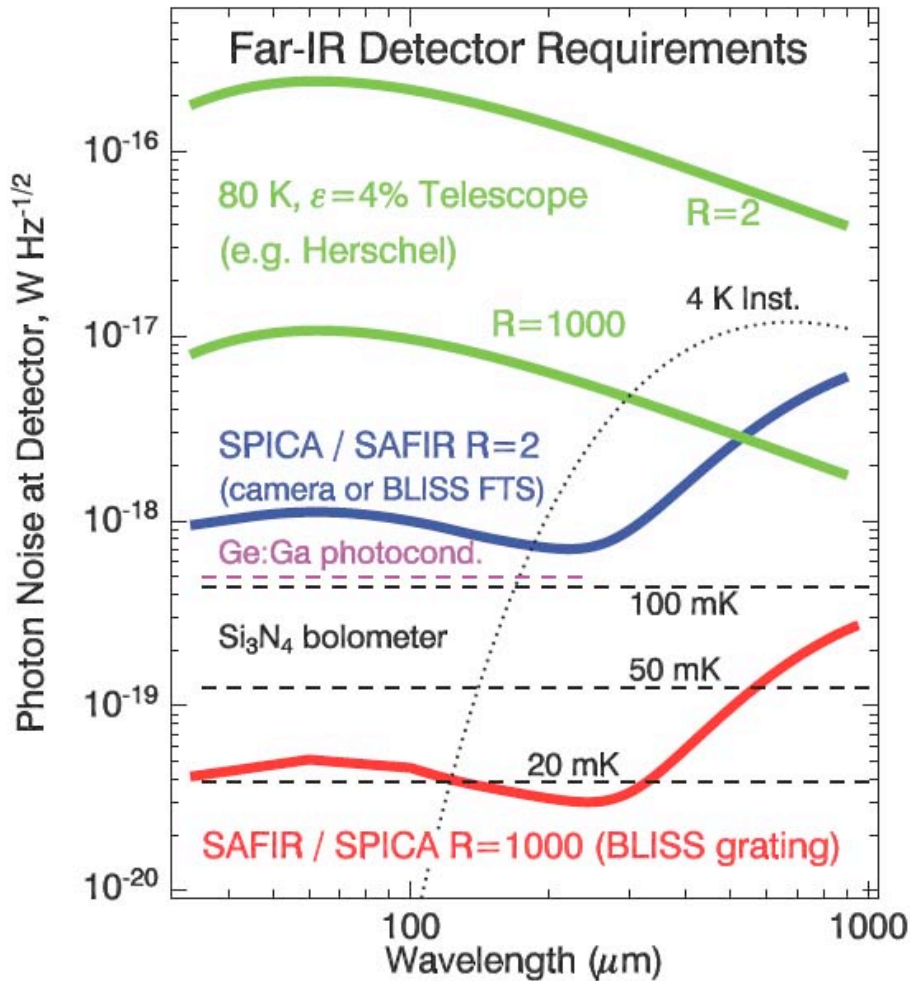


Figure 1: The photon noise at the detector in NEP units shows the requirements for reaching the background limit with a cold space telescope such as SAFIR or SPICA. For the continuum cameras or Fourier-transform type spectrometers, NEPs just under  $10^{-18}\ \text{W}\cdot\text{Hz}^{-1/2}$  are suitable – this is close to existing devices used for ground-based spectroscopy and CMB work. For moderate-resolution spectroscopy, however, devices with NEP of a few  $\times 10^{-20}\ \text{W}\cdot\text{Hz}^{-1/2}$  will be required. For comparison, the sensitivities appropriate for the higher-background Herschel telescope (green), and the photon noise contribution from many modes inside an instrument (4K,  $A\Omega = 3\ \text{mm}^2$ ) are plotted.

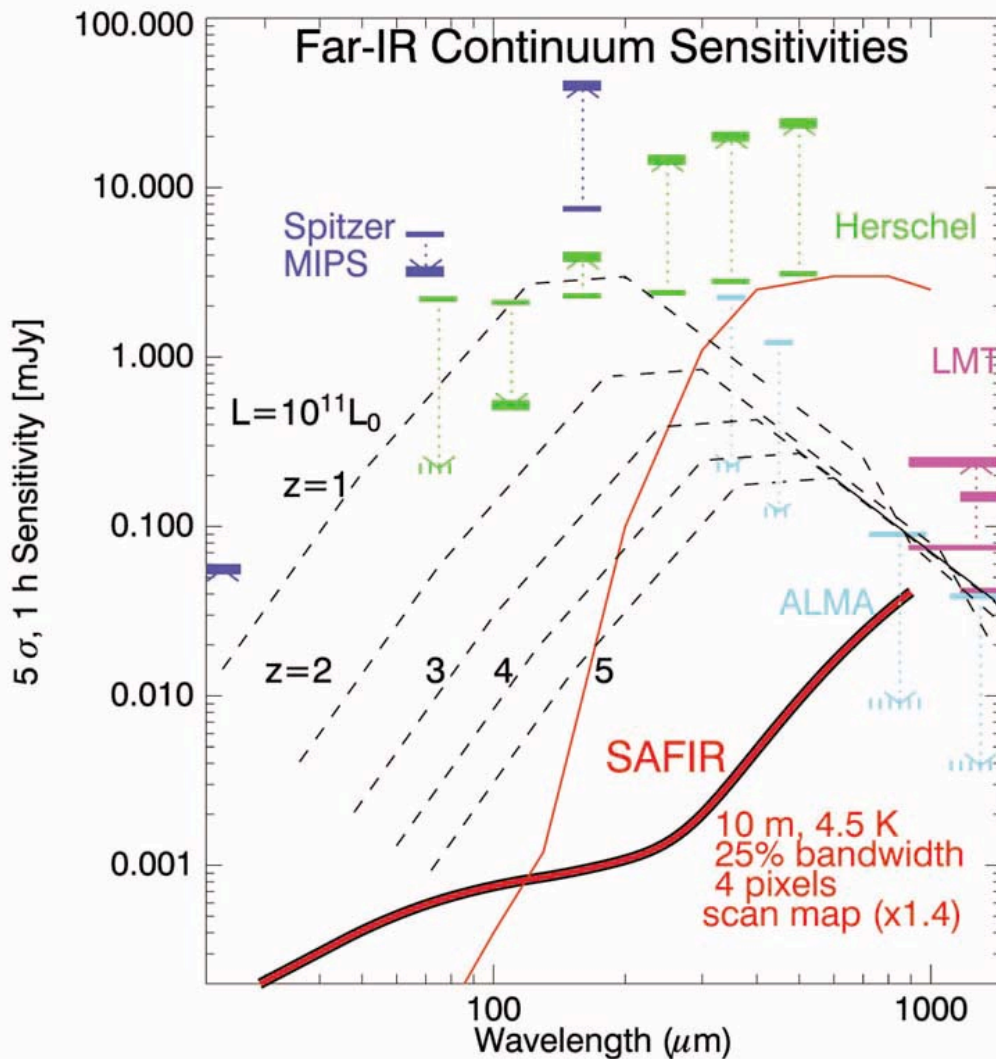


Figure 2: Raw sensitivity to point sources in the continuum for SAFIR and other far-IR platforms. Calculations for SAFIR assume an aperture efficiency of 75%, a camera with 50% throughput, and that 4 diffraction-limited pixels coupling both polarizations are used to extract the source flux. The Spitzer, Herschel, and ALMA sensitivities are taken from the instrument web pages. SOFIA has sensitivities comparable to those of Herschel. Values for the LMT (8) are estimated assuming a 50 m, 70  $\mu\text{m}$ , telescope with 0.8 mm of water vapor burden. For the platforms other than SAFIR, the thin horizontal lines correspond to the  $5\sigma$ , 1 hour raw sensitivity, while the thicker lines at the termination of the arrow represent the confusion limit. Upward arrows therefore imply an platform which is confusion limited in less than an hour, while downward arrows indicate that observations deeper than 1 hour will be fruitful. For SAFIR the thick red curve at the bottom of the plot is the raw sensitivity, while the thin red curve shows an estimate of the confusion limit. Confusion limits are taken to be 10 times the flux density at which there is one source per beam according to A.W. Blain et al. (2002 Physics Reports 369, 111.) Overplotted for reference (dashed black curves) are redshifted dusty galaxy SEDs appropriate for a  $10^{11} L_{\text{sun}}$  galaxy.

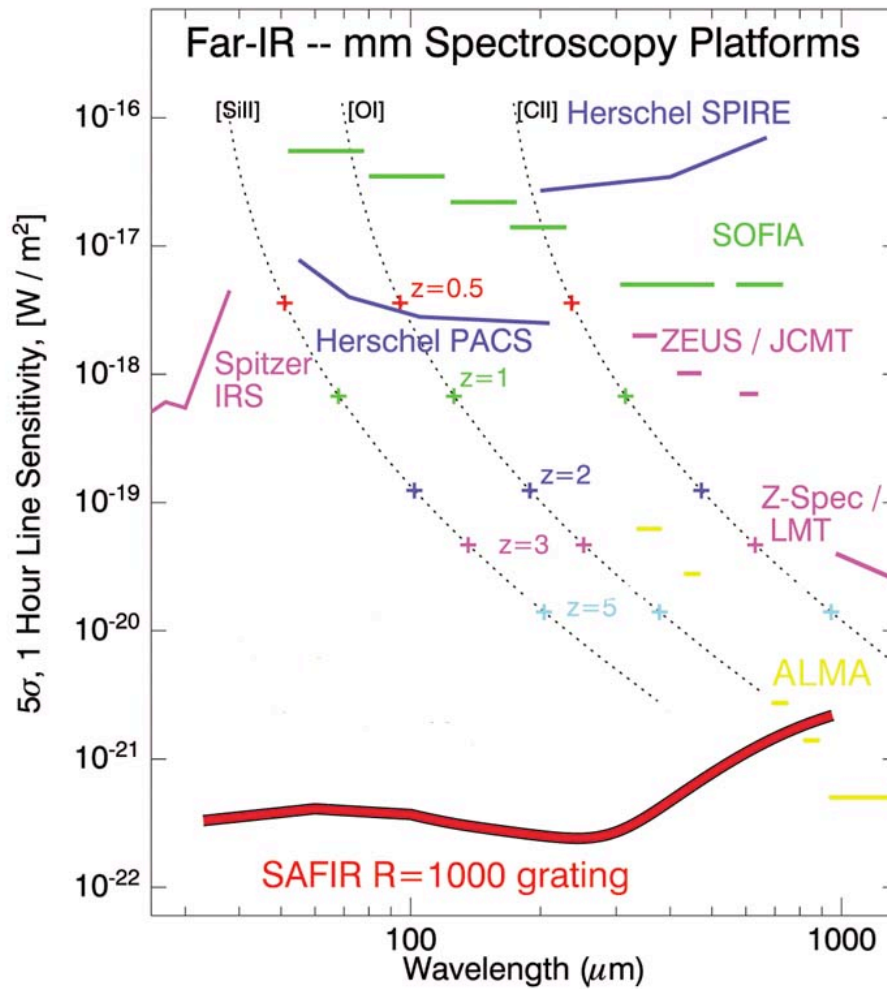


Figure 3: Sensitivity of far-IR spectroscopy platforms. Values for SOFIA, Herschel, Spitzer, and ALMA are taken from the instrument web pages. For ALMA, we assume a 300 km/s linewidth. Values for the ground-based ZEUS and Z-Spec instruments are taken from published values (T. Nikola et al: 2003 Proc. SPIE 4855, 88; B.J. Naylor, C.M. Bradford et al.: 2003 Proc. SPIE 4855, 239). The SAFIR curve (red, bottom) is calculated based on photon noise from the backgrounds, assuming a 10 m telescope with 75% aperture efficiency and 25% total instrument transmission in a single polarization, and a factor of two degradation for chopping. To show the degradation with finite detector NEP, the dashed black curve shows the sensitivity of a spectrometer on SAFIR with detectors with  $NEP = 3 \times 10^{-19} \text{ W-Hz}^{-1/2}$ . As a guide to the science capability, overplotted are spectral line intensities from a ULIRG ( $L = 10^{12} L_{\text{sun}}$ ) at various redshifts assuming a fractional line intensity of  $10^{-3}$ , and the current cosmological model ( $\text{vac} = 0.73$ ,  $\text{mat} = 0.27$ ,  $H_0 = 71$ ).